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SCIENTIFIC ASSOCIATION.

One Hundred and Twenty-second Regular Meeting, November 17, 1895.

Professor Remsen, President of the Association, in the Chair.

After a few remarks by the President, the following papers were presented and read:

1. The Discovery of Helium. By W. W. Randall.

Dr. Randall discussed the history of the discovery of helium and the sources from which the gas is derived. The methods employed for its purification were considered, as were also the apparatus with the aid of which the specific gravity was determined, and that by which the rate of propagation of sound-waves in the gas—and hence the ratio of the specific heats—could be ascertained. After reference to certain of the physical and chemical properties of helium, mention was made of the views which have been put forward to account for its peculiar behavior and to fix its place in the general system of the elements.

2. On the Spectrum of Clèveite Gas. By J. S. Ames.

The chief interest in the discovery of this new gas by Professor Ramsay lay, at first, in the fact that its spectrum gave the chromospheric line D_3, never before seen in the spectrum of a terrestrial gas, but which has been attributed to a solar element called "Helium." Further investigations by Crookes, Lockyer, Runge, Paschen and Vogel, have extended the knowledge of the spectrum of the gas, of its nature, and of its existence on the sun and bright stars. The spectra of many elements, *e. g.*, all the alkalies, consist of three series—one of strong lines and two of weaker ones, which may be single, double or triple, but which stop at identically the same wave-length in the more refrangible end of the spectrum. These series all obey the same law,

$$\frac{1}{\lambda} = A + \frac{B}{n^2} + \frac{C}{n^4}$$

where λ is the wave-length; A, B, C, are constants for any one series, and n is in turn each one of a series of integer numbers—1, 2, 3, 4, etc.

For elements of different atomic weights, the constants A, B, C, seem to vary in a regular manner. Runge and Paschen have been able to prove that there exist in the spectrum of Clèveite gas six series—two principal ones and four subordinate ones; and they have shown that, assuming the presence in the gas of *two* elements of molecular weights less than 6, the nature of the entire spectrum can be fully accounted for. The heavier of the two gases contains the D_3 line, and so must be called "Helium," while the other has received the provisional name "Parhelium."

In Young's observations of chromospheric lines, it is stated that some lines were always present, others less frequently; and Runge and Paschen have shown that this division of the lines corresponds accurately to their separation of the spectra of Helium and Parhelium. These same spectra occur in the light emitted by many of the fixed stars.

3. Solution and Diffusion of certain Metals in Mercury. By W. J. Humphreys.

The investigation, of which this is a summary, was begun with the object of determining the extent to which these phenomena differ, if at all, in this case from the solution and diffusion of non-metallic solids and liquids.

The method of investigation was to fill a vessel of constant cross section with pure mercury, put on its surface a freshly amalgamated piece of the metal to be examined, and after allowing it to stand a definite length of time in a place free from external disturbances and of fairly constant temperature, to remove from known depths below the surface samples of the amalgam and analyse them.

The metals examined were lead, tin, zinc, bismuth, copper and silver, and the results indicated that there is no essential difference between the solution and diffusion of these metals in mercury and the same phenomena in any other case.

Probably the most interesting results were those given by copper and silver, both of which dissolved to a much less extent than any of the other metals examined, but diffused more rapidly. At 28° C. the silver dissolved to the extent of only about one part in two thousand, and the copper to a still less extent—about three parts in a hundred thousand; while the rate of diffusion of the silver was about twenty millimeters per minute, approximately sixty times that of copper and fully six hundred that of zinc.

This investigation, of which the details will soon be published, was suggested to me by Dr. J. W. Mallet, F. R. S., of the University of Virginia, and carried out there under his supervision during the months of July, August and September, 1895.

The following papers of research were then presented and read by title:

1. Geometrical Multiplication of Series. By A. S. Chessin. (*Annals of Mathematics.*)
2. On Cauchy's Numbers. By A. S. Chessin. (*Annals of Mathematics.*)
3. On Divergent Series. By A. S. Chessin. (*Bull. Am. Math. Soc.*)
4. On a point of the Theory of Functions. By A. S. Chessin. (*Am. Jour. of Math.*)
5. Demonstration of the Existence of a Limit for Regular Sequences of Numbers. By A. S. Chessin. (See page 37.)
6. A New Classification of Infinite Series. By A. S. Chessin. (See page 39.)

An opportunity was then afforded those present to view the Helium spectra, the gas being in tubes brought from London by Dr. Randall.

One Hundred and Twenty-third Regular Meeting, December 19, 1895.

The President of the Association in the Chair.

The following papers were presented and read:

1. Theories of Color Sensation and of the Perception of Sound. By W. J. Mather.

In the last few years fresh discoveries and theories have aroused renewed interest in this subject. With reference to *sight*, the results obtained are of especial importance. On the anatomical side, Prof. Cajal has shown continuity of structure between the rods and cones and the nerve fibres of the retina; while the chief physiological interest has centered in the phenomena of color sensation. A study of the Young-Helmholtz and Hering theories prove their inability to account for many facts, while the theory of Mrs. Christine Ladd Franklin has been substantiated by several discoveries made since its proposal. Though hypothetical in its concept of a peculiar molecular cause, its explanation of the phenomena is quite scientific and it is therefore to be regarded as the more probable. With reference to *hearing*, the most important results have been obtained in the study of the equilibrium functions of the ears of fishes and other animals, substantiating the old idea of the static function of the labyrinths and showing that the otolithic structures serve as organs of orientation.

2. Recent Work on Impregnation in Flowering Plants. By J. E. Humphrey.

Until about four years ago, impregnation in flowering plants, which consists in the penetration of the pollen-tube to the egg-cell in the ovule, was known to take place only by the growth of the pollen-tube across the cavity of the ovary and through the micropyle left by the coats of the ovule. In 1891 Treub described impregnation in *Casuarina*, the Australian iron-wood, by the downward growth of the pollen-tube through the tissue of the ovary to the *chalaza*, or stalk of the ovule, and its upward growth through the body of the ovule to the egg-cell. In 1894 Miss Benson found the same thing to occur in several English catkin bearing plants, the hornbeam, alder, hazel, etc.

Nawaschin has just published the results of his detailed studies of the white birch, which agrees closely with the alder. In attempting to explain chalazal impregnation, he points out that the entire course of the pollen-tube of the *Gymnosperms* is through tissue. He thinks that in these primitive *Angiosperms*, the descendants of Gymnosperms, the tube has not yet acquired the ability to grow across open spaces, and therefore takes the indirect route which enables it to make its whole course through tissue. He also announces that the elm constitutes an intermediate form between those with chalazal and those with true micropylar impregnation.

Much work on this line is yet to be done, which may throw light on relationships among the flowering plants.

One Hundred and Twenty-fourth Regular Meeting, January 23, 1896.

The President of the Association in the Chair.

The following papers were presented and read:

1. The Temperature of the Earth's Interior. By G. K. Gilbert.

The speaker first pointed out the difficulty attending any investigation of the earth's interior, and stated that in the present condition of physical science all estimates of interior temperature are necessarily founded on questionable postulates. He then gave the results of a series of computations of the average temperature, each starting with a different group of postulates.

2. The Effect of Pressure on the Wave-lengths of Lines in the Spectra of certain Elements. By J. F. Mohler.

(See page 35 of this *Circular*.)

The following papers of research were then presented and read by title:

1. On Infinite Products. By A. S. Chessin. (See page 38.)
2. Additional Note on Divergent Series. By A. S. Chessin. (*Bull. Am. Math. Society.*)

One Hundred and Twenty-fifth Regular Meeting, February 20, 1896.

The President of the Association in the Chair.

The following papers were presented and read:

1. Reminiscences of James Dwight Dana. By Daniel C. Gilman.
2. Abnormal Cutaneous Sensibility. By Lewellys F. Barker.

SCIENTIFIC NOTES.

Effect of Pressure on the Wave-lengths of Lines in the Arc-spectra of certain Elements.

(Abstract of a paper by W. J. Humphreys and J. F. Mohler in The Astrophysical Journal for February, 1896, an account of which was given by Mr. Mohler to the University Scientific Association, January 23, 1896.)

The positions of the Fraunhofer lines have been regarded as constants of nature, subject to no possible change. Certain work, however, done in the Physical Laboratory of this University, made it appear that after all their positions may be in some measure a function of the conditions under which they are produced. This led us to examine the effects of pressure on arc-spectra.

In this examination we used a twenty-one and a half foot concave Rowland grating of 20,000 lines to the inch, mounted as described by Dr. Ames in the *Johns Hopkins Circular* of May, 1889. The arc was produced by a direct 110-volt current of any amperage desired, which probably amounted occasionally to fifty or more. The pressures were always obtained by pumping air into a cast-iron cylindrical vessel, which has at each end stuffing boxes through which pass insulated rods carrying carbons. A plane quartz window served to transmit light from the arc to the spectroscope. The pressures were indicated by a suitable gauge which could be read as often as desired, though the pressure never changed appreciably after the current was on a few seconds.

Nearly all the work was done in the second spectrum, the dispersion being a little more than one millimeter per Ångström unit. Some observations were taken directly with a micrometer eyepiece, but most of the results were obtained from photographs which were measured on a dividing engine especially constructed for this sort of work, and used in determining Rowland's table of standard wave-lengths.

The camera of the spectroscope, which holds a nineteen by a one-and-a-quarter inch plate, is arranged for making two successive exposures, one on a narrow strip along the middle of the entire plate and the other on the remaining portions. In front of the camera were arranged shutters that enabled us to follow our usual method of photographing, which was to expose about an inch of the middle strip of the plate to the sun, then the remainder of this strip to the arc under pressure, then the corresponding outer portions to the arc at atmospheric pressure, and finally the remaining parts to the sun. If plate or instrument had moved during the exposure, the solar lines would thus have shown it. Besides, the lines of the carbon bands were never appreciably shifted, and so these, some of which were found on nearly every plate, as well as the solar lines, enabled us to detect any disturbance of the apparatus during exposure. Unless stated to the contrary, a hole bored along the axis of the positive pole was filled with the material whose spectrum was to be examined.

Experimental Results.

Our first work was done with cadmium at atmospheric pressure. On varying the amount of material in the arc the width of the lines varied accordingly, and a large amount brought out clearly the reversals of the blue and green lines. The broadening was slightly unsymmetrical, being more towards the less refrangible or red portion of the spectrum, while the positions of the reversals coincided with those of the fine lines given by a small amount of material. Variation of strength of current did not appreciably affect the positions of the reversals of any lines examined, though the width of numerous lines increased, that of many unsymmetrically with increase of current. In the case of the aluminium lines of wave-lengths 3944 and 3961 the red side of the reversal was much stronger than the violet. Pressure was then applied to the arc containing cadmium and a decided shift in the positions of the lines was at once noticed. This shift was not due to unsymmetrical broadening, for it was possible to obtain fine sharp lines with and without pressure; nor was it due to the disappearance of one line and the appearance of another in a slightly different position, since it was often easy, while the pressure was being let off, to observe a line gradually change its position without alteration in width or other appearance. Such alterations, however, usually occurred unless special precautions were taken.

If, as is generally believed, the temperature of the electric arc is that of volatilizing carbon, it would be natural to suppose that it would rise with increasing pressure; but recent experiments by Wilson,[1] on the temperature of the arc under pressure, seem to show that it is lower at high pressure. In either case the shift might be due to change in temperature rather than pressure; and to test this point we used a long arc, perpendicular to the slit of the spectroscope, and with a heavy current exposed one part of a plate to the arc near the negative pole and the other to the arc near the positive pole. This method was adopted because results obtained by Wilson and Gray[2] indicate that the temperature of the negative pole is much lower than that of the positive. We could detect no change in the position of the lines, but this of course does not settle the question, as the temperature of the electric arc and how it varies from point to point is not known. The shift of the lines is certainly due to pressure, but whether directly or indirectly we can not say.

Over one hundred negatives were obtained and the shifts of some lines (those whose positions were well defined) of twenty-three elements measured in such manner that the accuracy for good lines was about three thousandths of an Ångström unit. That is, the positions of the lines were accurately determined to within about the one two thousandth part of the distance between the double D lines of sodium.

The negatives show that the effect of pressure is to broaden the lines and to bring out the reversals. However this is not always the case, as it is possible to obtain with cadmium, for example, lines of about the same width with and without pressure, while in the case of some elements, as platinum and osmium, the lines are often narrower at high than at atmospheric pressure. The lines of the carbon bands are brought out more strongly by pressure, but, as already stated, they never show much if any shift, which fact furnishes conclusive evidence that the shift of other lines is not due to a disturbance of the apparatus, since they were all photographed simultaneously on the same plate, the lines of the carbon bands never being appreciably displaced while those of other elements are.

This displacement, or shift, varies greatly for different elements, but in the case of any one, with a single exception to be mentioned below, it is approximately proportional to the wave-length. The most conclusive evidence of this proportionality is furnished by lines of different orders of spectra that appear on the same plate. Thus ultra-violet lines of the third order are often found on the same plate with lines of the second of longer wave-length, but the measured shifts of the lines of the same element are approximately the same; and since the wave-length of a line of the third order is to that of one of the second that occurs at the same place as two to three, while the dispersion in the third order is to that in the second as three to two, it follows that constancy of measured shift means that it is proportional to wave-length. For the sake of comparison it was found advisable to reduce the shifts of all lines to what they would be at wave-length 4000, in the neighborhood of which most of our work was done. This reduction consisted, in accordance with the above, in multiplying the measured shift of any line by the ratio of 4000 to its wave-length. The exception referred to above is that of calcium. The lines H and K among others, shift only about half as much as *g* and the group at wave-length 5600. That *g* should differ in this respect from H and K is not very surprising, since it is known to differ from them in many other respects. According to Lockyer[3] it is the "longest line" of the calcium spectrum; that is, it occurs at the greatest range of temperature, even at that of an ordinary flame, while the H and K do not appear at temperatures much below that of the electric arc.

All our measurements show that the shifts are invariably towards the less refrangible or red end of the spectrum, and that they are directly proportional, not only to the wave-lengths, but also to the excess of pressure (at least to fourteen and one-half atmospheres, the limit of our work) above one atmosphere. It was found impossible, owing to the minuteness of the shifts per atmosphere and to the fact that our apparatus was not designed for vacuum work, to determine whether the same law holds for very low pressures.

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[1] Proc. R. Soc., May 30, 1895.

[2] Proc. R. Soc., November 24, 1894.

[3] Chemistry of the Sun.

It should be stated that in some cases the values obtained for the shifts may be due in a measure to unsymmetrical broadening; but this has certainly not led to much error, since, as already stated, only those lines were used which could be fairly accurately measured—that is, those which were either comparatively narrow or else reversed.

A very satisfactory negative was obtained from an arc formed between a carbon negative and an iron positive pole. The pressures used were one and ten atmospheres. This photograph shows that, when the lines are close together, the effect of pressure is to form a practically continuous spectrum with only reversed lines, thus in a measure approaching the condition of the solar spectrum.

THEORETICAL RELATIONS.

In considering the numerical values of the shifts of the lines of different elements certain mathematical relations were discovered.

It was first noticed that, by forming for various elements the product of the cube root of the atomic volume and the coefficient of linear expansion of the substance in the solid form, certain numbers are obtained whose ratios are the same as those of the shifts for the respective elements. The atomic volume of any element is the ratio of its atomic weight to its density at any standard temperature; and the cube root was taken because the wave-length of any series of waves produced by a body varies directly as its linear dimensions. In our figures the atomic volume and the coefficient of expansion both refer to 40° C., and the shift is that which corresponds to wave-length 4000 and pressure of twelve atmospheres. The agreement of the two sets of figures may be seen in Table I, columns five and six.

Raoul Pictet[1] used a similar expression in his formula for deducing the melting points of the metals. He finds that the continued product of the absolute temperature of the melting point, the linear coefficient of expansion, and the cube root of the atomic volume is nearly the same for all the metals except bismuth.

Table I shows the connection of our work with that of M. Pictet, the eighth column giving the quotient of a constant, *i. e.*, 48600 divided by the absolute temperature of the melting point. The number 48600 was taken so as to reduce his results to numbers comparable with the shift at twelve atmospheres pressure, that for iron being made to coincide with the "theoretical value" $\alpha\sqrt[3]{V}$. This table shows that the shift is about as near the "theoretical value" as the melting point, and consequently the product of the shift by the absolute temperature of the melting point is (with the exception of bismuth) nearly constant, or that the shift is inversely proportioned to the absolute temperature of the melting point.

According to these considerations the shift of the lines of mercury ought to be very large. A number of attempts were made to photograph the mercury lines, but without good results, as the lines, though showing a large displacement, were too hazy and diffuse to admit of accurate measurement.

TABLE I.

	W	V	α	$\alpha\sqrt[3]{V}$	S	T	$\frac{48600}{T}$
	......	......	.0000	......	......	......	......
Fe	55.9	7.2	1210	23.3	25	2080	23.3
Ni	58.6	6.7	1279	24.	28	1870	26.5
Co	58.6	6.9	1236	23.6	24	2070	24.
Mn	54.8	6.9	......	......	33	2170	22.9
Cr	52.45	7.7	......	......	26	......	......
Bi	207.3	21.1	1621	44.7	49	538	90.3
Pb	206.4	18.1	2924	76.9	60	605	80.3
Sn	118.8	16.3	2234	58.4	55	503	96.6
Tl	203.7	17.2	3121	78.	61	563	86.3
In	113.6	15.3	4170	103.5	88	449	108.3
Al	27.04	10.6	2313	50.6	55	1123	43.3
Pt	194.3	9.1	0899	18.5	20	2050	23.7
Os	190.3	8.5	0657	13.4	17	2770	17.5
Rb	104.1	8.6	0850	17.4	30	2270	21.4
Pd	106.35	9.2	1176	24.7	33	1775	27.4
Y	88.9	......	......	......	15	......	......
B	10.9	4.	......	......	49	......	......
C	11.97	3.6	0118	2.7	0	......	......
Ag	107.7	10.2	1921	42.2	38	1230	39.5
Cu	63.18	7.1	1678	32.5	33	1330	36.5
Ca	39.91	25.4	......	......	27	......	......
Ca	39.91	25.4	......	......	54	....	......
Zn	65.10	9.1	2918	61.2	57	676	71.9
Cd	111.7	12.9	3069	75.6	80	593	82.

DESCRIPTION OF TABLE I.

The column marked W gives the atomic weights of the elements. Under V is given the atomic volume at 40° C., and under α the linear coefficient of expansion. In all cases this coefficient is taken from Fizeau's results at 40° C. The column marked $\alpha\sqrt[3]{V}$ gives the product of the linear coefficient of expansion by the cube root of the atomic volume multiplied by 10^6. Under S is the reduced shift in thousandths of an Ångström unit at twelve atmospheres pressure. The column marked T gives the absolute temperature of the melting point, and the last column, marked $\frac{48600}{T}$ gives the quotient indicated.

The values under W, V and T are taken from Nernst, Theoretical Chemistry.

Besides the relations already described, a connection between the atomic weights and the shifts of the corresponding elements was also found. While this connection is not perfect, it nevertheless seems altogether too general to be merely accidental. The relation is as follows: Given the shifts of the lines of any element, the shifts of those of other elements belonging to the same Mendelejeff group, can be found on the supposition that they are to each other as the cube roots of the respective atomic weights. This is shown in the first part of Table II, in which the standard shifts, assumed to be correct, have all been determined by experiment and tabulated in Table I.

Each horizontal row of Table II shows the comparison between the observed shift of the lines of one element and that calculated for them from the observed shift of the lines of another element of the same group.

It will be observed that the calculated values agree almost exactly with the observed except in the case of boron, which, besides differing from the other elements in being non-metallic, furnished only two lines that could be measured, and even these were not good.

TABLE II.

Showing shifts in thousandths of an Ångström unit for twelve atmospheres and wave-length 4000.

STANDARD.			CALCULATED.	OBSERVED.
Cu	33	Ag	39	38
Cd	80	Zn	66	57
Cd	80	Ca (*g*)	56	54
Al	55	In	89	88
Al	55	B	36	49
Sn	55	Pb	66	60
Co	24	Rb	29	30
Ni	28	Pd	34	33
Cd	80	Ca (H and K)	28	27
Al	55	Tl	54	61
Al	55	Y	41	15
Fe	25	Os	19	17
Ni	28	Pt	21	20

Beyond the fact that atoms are exceedingly minute scarcely anything is known of their actual size, nor is anything at all definitely known as to their density, and for all we know the atoms of the same group may have either the same or different densities. If they have the same density then (unless they differ in shape) their linear dimensions are to each other as the cube roots of their atomic weights; and consequently our measurements would mean that the shifts of elements of the same group bear the same ratios to each other as the linear dimensions of their atoms. Or, on the other hand, if it be accepted that the shifts are directly proportional to the linear dimensions of the vibrating particles, then it would follow that the atomic density is constant for the elements of any group.

The lines of a few elements, yttrium, osmium, platinum, thallium and calcium (H and K) do not give shifts which at first seem to agree with this rule. If, however, it be assumed that these substances dissociate in the arc into eight equal parts (possibly dividing along three intersecting planes of symmetry and thus for many configurations giving the same shape to the parts as that of the original and reducing the linear dimensions by half) then their observed shifts and those calculated, using one-eighth the atomic weights, agree within experimental errors, except in the case of yttrium.

---

[1] C. R., 88, 855, 1879.

While the shifts of the lines due to elements of different groups differ from each other greatly, it is nevertheless possible, in most cases, at least, to pass from one group to another by means of the ratios of the coefficients of expansion of elements of different groups but of the same series. Thus within experimental errors the ratio of the coefficients of expansion of copper to zinc, of silver to indium and tin, of cadmium to indium and tin, and of aluminium to iron nickel and cobolt is in every case equal to the corresponding ratio of shifts.

Though the lines of the carbon bands do not shift appreciably, the single carbon line at wave length 2478 shifts, according to measurements made on several plates, about forty-six thousandths of an Ångström unit at a pressure of twelve atmospheres.

This investigation was conducted under the supervision of the directors of the Physical Laboratory, Drs. Rowland and Ames. We would especially acknowledge our indebtedness to Dr. Ames for his assistance, as many of the points brought out are the direct results of his suggestions. Our thanks are also due to Mr. Jewell for assistance in the choice and measurement of the lines, and to Dr. Tatnall for kindly sending us his measurements, not yet published, of the palladium lines.

Physical Laboratory of the Johns Hopkins University.

Demonstration of the Existence of a Limit for Regular Sequences of Rational Numbers. By A. S. Chessin.

Before taking up this demonstration let us briefly recall the definition and properties of regular sequences of rational numbers.

Definition I.—A sequence of rational numbers which may be indefinitely extended according to a determinate law.

$$(1) \qquad \gamma_1,\ \gamma_2,\ \gamma_3,\ \ldots,\ \gamma_m,\ \ldots$$

is said to be regular if all its terms remain in absolute amount less than a finite determinate number, and if at the same time a number m can be found such that the difference

$$|\gamma_{m+n} - \gamma_m| \ (n = 1, 2, 3, \ldots)$$

can be made less than an arbitrarily small number ϵ.

Definition II.—Given a regular sequence (1), let q_κ be the quantity represented by the number γ_κ. If a quantity Q exists such that

$$|Q - q_{m+n}| \ (n = 1, 2, 3, \ldots)$$

can be made less than an arbitrarily small quantity, this quantity Q will be called the limit of the sequence

$$q_1,\ q_2,\ q_3,\ \ldots,\ q_m,\ \ldots$$

If the quantity Q can be represented by a number Γ we have the following:

Definition III.—Given a regular sequence (1), if a number Γ exists such that

$$|\Gamma - \gamma_{m+n}| \ (n = 1, 2, 3, \ldots)$$

can be made less than an arbitrarily small number ϵ, this number Γ will be called the limit of the sequence (1), and to express it the notation $\Gamma = \lim. (\gamma)$ will be used.

Theorem I.—A sequence of rational numbers

$$\gamma_1,\ \gamma_2,\ \gamma_3,\ \ldots,\ \gamma_m,\ \ldots$$

in which

$$|\gamma_{m+n}| \ (n = 1, 2, 3, \ldots)$$

can be made less than an arbitrarily small number is regular and has zero for its limit.

That this sequence is regular is obvious. On the other hand,

$$|0 - \gamma_{m+n}| = |\gamma_{m+n}|$$

can be made less than ϵ, hence $\lim. (\gamma) = 0$.

Theorem II.—In every regular sequence

$$\gamma_1,\ \gamma_2,\ \gamma_3,\ \ldots,\ \gamma_m,\ \ldots$$

which has not zero for limit, the terms from a certain place onwards are either all positive or all negative.

In fact, suppose that whatever be m we can always find two numbers in our sequence γ_{m+n_1} and γ_{m+n_2} which differ in sign. The difference $|\gamma_{m+n_2} - \gamma_{m+n_1}|$ is not less than the greater of the numbers $|\gamma_{m+n_2}|$ and $|\gamma_{m+n_1}|$. Now, since there is a value of m such that $|\gamma_{m+n_2} - \gamma_{m+n_1}| < \epsilon$, it follows that for this value of m both $|\gamma_{m+n_1}|$ and $|\gamma_{m+n_2}|$ are $< \epsilon$.

If then γ_{m+n} be any term of the given sequence beyond γ_{m+n_1} and γ_{m+n_2} we necessarily conclude that

$$|\gamma_{m+n}| < \epsilon \begin{pmatrix} n > n_1 \\ n > n_2 \end{pmatrix}$$

contrary to the hypothesis that $\lim. (\gamma) \neq 0$.

Theorem III.—If in a sequence of rational numbers the terms from a certain place onwards continuously increase, remaining in absolute amount less than a finite determinate number, or if the terms from a certain place onwards continuously decrease, but do not in absolute amount fall below a finite determinate number, then the given sequence is regular. The demonstration of this theorem is very simple.

After these preliminaries we can take up the proposed problem.

Since the existence of a limit has been proved for sequences such as considered in Theorem I, we may assume that the limits of the regular sequences which we now consider, if they exist, are different from zero. Then, by Theorem II, we may assume that we have only positive or only negative terms in our sequences, for we only need to begin these sequences from the place from which onwards this condition is satisfied.

To fix the ideas, let us consider a regular sequence

$$(2) \qquad \alpha_1,\ \alpha_2,\ \alpha_3,\ \ldots,\ \alpha_m,\ \ldots$$

of only positive rational numbers.

Two cases present themselves: either the terms of this sequence will form a certain place μ onwards continuously increase or continuously decrease; or there is no place from which onwards the terms of the given sequence continuously increase or continuously decrease.

Case I.

$$\alpha_\mu < \alpha_{\mu+1} < \alpha_{\mu+2} < \ldots < \alpha_{\mu+n} < \ldots$$

The sequence (α) being regular we have $\alpha_{\mu+n} < N \ (n = 1, 2, 3, \ldots)$, where N is a finite determinate number.

Consider now the ratios

$$(3) \qquad \frac{\alpha_\mu}{\alpha_{\mu+1}},\ \frac{\alpha_{\mu+1}}{\alpha_{\mu+2}},\ \ldots,\ \frac{\alpha_{\mu+n-1}}{\alpha_{\mu+n}},\ \ldots$$

We have from a certain place m onwards (m may be $\gtreqless \mu$)

$$\alpha_{m+n} - \alpha_m < \epsilon \ (n = 1, 2, 3, \ldots);$$

i. e., the terms of the sequence (3) remaining < 1 differ from it by an arbitrarily small number. We may write that

$$\frac{\alpha_{m+n}}{\alpha_{m+n+1}} = 1 - \epsilon_n \ (n = 1, 2, 3, \ldots)$$
$$0 < \epsilon_n < \epsilon;$$

i. e. $\lim. (\epsilon) = 0$.

We now form a new sequence,

$$\beta_1,\ \beta_2,\ \beta_3,\ \ldots,\ \beta_m,\ \ldots$$

of only positive terms and such that from the place m onwards

$$(4) \qquad \beta_{m+n+1} < (1 - \epsilon_n)\,\beta_{m+n} - \epsilon_n \ (n = 1, 2, 3, \ldots)$$

The sequence (α) being given, the numbers ϵ_n are quite determinate and remains to see whether such positive numbers β_κ can be found as to satisfy the inequality (4).

Let us put

$$(5) \qquad \beta_{m+n} = \frac{\alpha_m}{\alpha_{m+n}}\,\beta_m - \frac{\alpha_{m+n} - \alpha_m}{\alpha_m}$$

Then

$$\begin{aligned} \beta_{m+n+1} &= \frac{\alpha_m}{\alpha_{m+n+1}}\,\beta_m \quad \frac{\alpha_{m+n+1} - \alpha_m}{\alpha_m} \\ &= (1 - \epsilon_n)\left(\beta_{m+n} + \frac{\alpha_{m+n} - \alpha_m}{\alpha_m}\right) - \frac{\alpha_{m+n+1} - \alpha_m}{\alpha_m} \\ &< (1 - \epsilon_n)\,\beta_{m+n} - \frac{\alpha_{m+n+1} - \alpha_m}{\alpha_m} \end{aligned}$$

But

$$\frac{a_{m+n+1} - a_{m+n}}{a_m} > \frac{a_{m+n+1} - a_{m+n}}{a_{m+n+1}} \ i.\,e. > \epsilon_n$$

Hence

$$\beta_{m+n+1} < (1 - \epsilon_n)\,\beta_{m+n} - \epsilon_n$$

i. e., the numbers β_{m+n} defined by formula (5) satisfy the inequality (4). Remains to determine them in such a manner as to make them all positive. We must then have

$$(6) \qquad \beta_m > \frac{a_{m+n}}{a_m} \cdot \frac{a_{m+n} - a_m}{a_m}$$

But, whatever be n, we always have

$$a_{m+n} - a_m < \epsilon$$

If, therefore, we take for β_m the value

$$\beta_m = \frac{N\epsilon}{{a_m}^2}$$

the inequality (6) will be satisfied and by (5) all the numbers β_{m+n} ($n =$ I, 2., 3, ...) will be positive.

We now form a new sequence :

$$(7) \qquad \gamma_1, \gamma_2, \gamma_3, \ldots., \gamma_m, \ldots.$$

where

$$\gamma_\kappa = (1 + \beta_\kappa)\,a_\kappa.$$

From the place m onwards the terms of this sequence will continuously decrease. In fact by (4), whatever be n,

$$(1 + \beta_{m+n+1})\,a_{m+n+1} < (1 + \beta_{m+n})\,a_{m+n}.$$

Moreover, since the numbers $\beta_{m+n} > 0$, we have $\gamma_{m+n} > a_{m+n}$ whatever be n. The sequence (7) is therefore by Theorem III. regular.

Thus we have two regular sequences—

$$a_1, a_2, a_3, \ldots., a_m, \ldots.$$
$$\gamma_1, \gamma_2, \gamma_3, \ldots., \gamma_m, \ldots.,$$

the first composed from a certain place onwards of only increasing terms, the second from the same place onwards of only decreasing terms.

Let us now define a quantity Q by the following property :

Quantity represented by the number a_{m+n} $< Q <$ Quantity represented by the number γ_{m+n}

whatever be n. The existence of such a quantity is conceivable *à priori*. We have on the other hand

$$\gamma_{m+n} - a_{m+n} = a_{m+n}\,\beta_{m+n} < a_m\,\beta_m$$
$$i.\,e. < \frac{N}{a_m}\,\epsilon$$

therefore the difference $\gamma_{m+n} - a_{m+n}$ can be made less than an arbitrarily small number, whatever be n. Accordingly the difference between the quantities which these numbers represent can be made less than the arbitrarily small quantity represented by the number ϵ. A fortiori the difference between the quantities represented by the numbers a_{m+n} and γ_{m+n} and the quantity Q can be made less than the arbitrarily small quantity represented by the number ϵ. But *this result is nothing else than the statement that the sequences of the quantities represented by the numbers* a_{m+n} and γ_{m+n} *have a common limit as defined at the beginning of this paper*, namely, the quantity Q. The question of representation of this quantity by a symbol or number constitutes another problem. Suppose that a number N is defined so as to represent this quantity Q; then we have proved that $N = lim\,(a) = lim\,(\gamma)$.

If the terms a_κ instead of continually increasing, decreased continuously from a certain place onwards, the demonstration would be quite similar to the one just given.

CASE II.

We consider again the regular sequence of positive numbers (2) only now we have

$$\ldots. > a_\mu < a_{\mu+1} < \ldots. < a_{\mu_1} > a_{\mu_1+1} > \ldots. > a_{\mu_2} < a_{\mu_2+1} < \ldots.$$

Two cases present themselves here: either (1) we can find no number a_ν in our sequence such that it be greater than any other term of the same, or (2) we can find such a number a_ν.

In the first case we can find numbers $a_\nu < a_{\nu_1} < a_{\nu_2} < \ldots.$, *i. e.* we can form a regular sequence (a_ν) such as considered in Case I; but then the existence of the limit is proved.

In the second case we leave out for a moment the term a_ν and reason with regard to the remaining terms of the sequence in a similar way. We shall either prove the existence of a limit or find a number $a_{\nu_1} < a_\nu$ but greater than any other term of the sequence. Continuing this reasoning in the same way we shall either prove the existence of a limit or form a regular sequence

$$a_\nu,\ a_{\nu_1},\ a_{\nu_2},\ \ldots.$$

with continuously decreasing terms, *i. e.* reduce again this case to the one considered under Case I. Hence the existence of a limit is proved in all cases, and the proposed problem is thus solved.

On Infinite Products. By A. S. Chessin.

The proposition that absolute convergence of infinite products implies unconditional convergence and *vice versa* is generally proved by comparing infinite products with certain infinite series. The usual way is to take the log of the infinite product, a method the deficiency of which is obvious for more than one reason. It is true that Pringsheim (*Math. An.*, 33, p. 119) gave a demonstration of this proposition without the use of the log, but his proof, which is rigorous and elegant, is also based on the theory of infinite series. I will endeavor to prove the proposition independently of any considerations of infinite series, restricting myself in this note to infinite products with real factors.

Theorem I.—Absolutely convergent infinite products of real factors are unconditionally convergent.

Let $\prod\limits_{\kappa=1}^{\kappa=\infty} (1 + a_\kappa)$ be the given infinite product. If P be its value, we may assume without loss of generality that $P > 0$ since all the factors from a certain place onwards are necessarily positive on account of $\lim\,(a_\kappa) = 0$. We denote by p_n the product of the first n factors of the given infinite product, *i. e.*,

$$p_n = \prod_{\kappa=1}^{\kappa=n} (1 + a_\kappa)$$

and by p''_n the same product after $|a_\kappa|$ had been substituted for a_κ, *i. e.*,

$$p''_n = \prod_{\kappa=1}^{\kappa=n} (1 + |a_\kappa|)$$

By hypothesis an arbitrarily great but finite number n can be found such that

$$(1) \qquad \frac{p''_{n+\mu}}{p''_n} - 1 < \epsilon$$
$$\mu = 0, 1, 2, 3, \ldots.$$

where ϵ is an arbitrarily small but finite positive number.

Let now $p'_{n'}$ denote the product of the first n' factors of the given infinite product after its factors had been subjected to a commutative change. We can always choose n' sufficiently great to have all the factors of p_n included among those of $p'_{n'}$. With a value of n' so chosen $p'_{n'} = p_n$ multiplied by a number of factors all between $1 + a_{n+1}$ and a certain factor $1 + a_{n+\mu+1}$. Therefore,

$$p'_{n'} \leqq p_n\,(1 + |a_{n+1}|)\,(1 + |a_{n+2}|) \ldots (1 + |a_{n+\mu}|),$$

or, what is the same thing,

$$p'_{n'} \leqq p_n\,\frac{p''_{n+\mu}}{p''_n}.$$

This inequality becomes

$$p'_{n'} < p_n\,(1 + \epsilon)$$

on account of (1), *i. e.*,

$$\lim\,(p'_{n'}) = \lim\,(p_n)$$

q. e. d.

Theorem II.—Semi-convergent infinite products of real factors are conditionally convergent.

Let us denote by a_κ the positive, by $-b_\kappa$ the negative numbers a_κ which enter into the expression of the given infinite product; let again

$$p_n = \prod_{\kappa=1}^{\kappa=n} (1 + a_\kappa)$$

while we will denote by $p'_{n'}$ and $p''_{n''}$ partial products contained in p_n, the first composed of factors $(1 + a_\kappa)$, the second of factors $(1 - b_\kappa)$; *i. e.*

$$p_n = p'_{n'}\, p''_{n''}. \tag{2}$$

$$p'_{n'} = \prod_{\kappa=1}^{\kappa=n'} (1 + a_\kappa) \tag{3}$$

$$p''_{n''} = \prod_{\kappa=1}^{\kappa=n''} (1 - b_\kappa) \tag{4}$$

$$n' + n'' = n$$

Again we may assume without loss of generality that $p'_{n'}$, $p''_{n''}$ and p_n are all positive numbers. We will show now that

$$\lim\, (p'_{n'}) = \infty \tag{5}$$

$$\lim\, (p''_{n''}) = 0. \tag{6}$$

To this end we first prove that $p'_{n'}$ and $p''_{n''}$ can not tend both to finite and determinate limits (not zero) independently of each other. In fact, if this were the case we would find on account of the inequality

$$0 < (1 + b_{n+1})\,(1 + b_{n+2}) \ldots (1 + b_{n+\mu}) < \frac{1}{(1 - b_{n+1}) \ldots (1 - b_{n+\mu})}$$

that the infinite product

$$\prod_{\kappa=1}^{\kappa=\infty} (1 + b_\kappa) \quad .$$

were convergent. If then $(p''_{n''})$ were to denote $p''_{n''}$ after the change of $-b_\kappa$ into $+b_\kappa$, and (p_n) the corresponding expression of p_n, we should have had

$$(p_n) = p'_{n'}\,(p''_{n''})$$

and

$$\lim \left[(p_n)\right] = \prod_{\kappa=1}^{\kappa=\infty} (1 + |\, a_\kappa \,|) = \left\{ \begin{array}{l} \text{A finite determinate number} \\ \qquad \text{other than zero.} \end{array} \right.$$

But that would mean that the given infinite product is absolutely convergent, which is contrary to the statement of this theorem.

On the other hand it is not possible to have $\lim\, (p'_{n'}) = \infty$ and $\lim\, (p''_{n''})$ = a finite and determinate number (not zero); or to have $\lim\, (p'_{n'})$ = a finite and determinate number (not zero) and $\lim\, (p''_{n''}) = 0$; because then the given infinite product would be divergent, contrary to the statement of this theorem. Hence we must have $\lim\, (p''_{n''}) = 0$.

After these preliminaries let N_1 and N_2 be two positive numbers and let $N_1 > N_2$. We can take the number n' in (3) sufficiently great to have $p'_{n'} > N_2$. Let n_1 be this value of n', so that

$$p'_{n_1} > N_2$$

We now take n'' in (4) sufficiently great to have $p'_{n_1}\, p''_{n''} < N_1$; let n_2 be this value of n'', so that

$$p'_{n_1}\; p''_{n_2} < N_1$$

Next we take a number n_3 of factors in (3) sufficiently great to have

$$p'_{n_1}\, p''_{n_2} \frac{p'_{n_3}}{p'_{n_1}} > N_2$$

and again a number n_4 of factors in (4) sufficiently great to have

$$p'_{n_1}\, p''_{n_2} \frac{p'_{n_3}}{p'_{n_1}}\, \frac{p''_{n_4}}{p''_{n_2}} < N_1$$

and so on. With this change of the order in the succession of factors the product p_n will oscillate between the numbers N_1 and N_2 and on account of $\lim\limits_{\kappa=\infty} (a_\kappa) = 0$ or $\lim\limits_{\kappa=\infty} (1 + a_\kappa) = 1$ the interval of these oscillations will approach indefinitely to the value $N_1 - N_2$, while we can by taking a proper number of terms make the product p_n differ from either N_1 or N_2 by an arbitrarily small quantity. The change in the order of the factors has therefore changed the semi-convergent product into an oscillating, *i.e.*, divergent one. q. e. d.

Had we chosen $N_1 = N_2$ the interval $N_1 - N_2$ would be zero and we obtain the following:

Theorem III.—A semi-convergent infinite product of real factors $\prod\limits_{1}^{\infty} (1 + a_\kappa)$ *can by a proper commutative change in the order of its factors be made to converge to any arbitrarily chosen number N,* this number being chosen positive or negative according as $\prod\limits_{1}^{\infty} (1 + a_\kappa)$ is positive or negative.

This is the proposition for infinite products which corresponds to the well known theorem of Rieman with regard to semi-convergent series.

A New Classification of Infinite Series. By A. S. Chessin.

Infinite series of numbers are usually classified either into convergent and divergent series or into convergent, oscillating and divergent series—a distinction being made between absolutely convergent and semi-convergent series. It is well known that absolute convergence implies unconditional convergence and *vice versa*. But as to semi-convergence the word "conditional" is usually applied only with regard to the commutative law. I have shown in a note published in the *Bull. of the Amer. Math. Society* (Dec., 1895) that *oscillating series are also conditionally convergent*,[1] namely: every oscillating series can by a proper association of its terms be made convergent. The following classification of infinite series of numbers is therefore proposed.

Class I.—Convergent infinite series which are subject to the associative, commutative and distributive laws. These are the absolutely or unconditionally convergent series.

Class II.—Infinite series which are subject to the associative law without restriction, being only partly subject to the distributive law,[2] and not subject at all to the commutative. These are the semi-convergent series. They are conditionally convergent.

Class III.—Infinite series which are not subject to any of the three laws named. These are the oscillating series. They are conditionally convergent.

Class IV.—Infinite series which are subject to the associative, commutative and distributive laws, but are not convergent. They are unconditionally divergent and tend to infinity in a determinate way.

On the Representation of Imaginary Plane Curves.

A Letter to Professor Craig by René de Saussure.

Washington, *January 10th*, 1896.

Professor Thomas Craig,
Johns Hopkins University.

Dear Sir.—Having obtained lately certain results concerning the representation of imaginary plane curves, which results seem to me to have some value, I wish to submit them to you briefly.

Each point of an imaginary plane can be represented by a straight line in our three dimensional space, for the position of such a straight line depends upon four real parameters x_1, x_2, y_1, y_2, which are equivalent to two complex parameters: $x = x_1 + ix_2$, $y = y_1 + iy_2$. The straight line x, y, can then be regarded as the representation of the imaginary point x, y, situated in a fixed imaginary plane. Since an imaginary plane curve has a double infinity of points, its representation in the ruled space will be a congruence of straight lines.

I found, however, that such a representation is known, but as there are an infinity of ways to determine a straight line by four parameters, and as the representation of the same imaginary plane curve $y = f(x)$ will be entirely different according to the system of coördinates used, I have tried to establish a correspondence between each point of the imaginary plane and each straight line of the real space, which is independent of any system of coördinates. This I arrive at by the following theorem:

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[1] In the note referred to, as well as in the present note, only such series are considered the general term of which u_n tends to zero as n tends to infinity.

[2] The product of two convergent infinite series is convergent if at least one of them belongs to the Class I. But the product of two series of the Class II. may belong to the Class II. or to the Class III.

Let I be a fixed imaginary plane in space; through any point m of the plane I passes a real straight line M and only one. To prove this, draw a real plane R parallel to the plane I at the distance $i=\sqrt{-1}$, and take two rectangular axes of coördinates ox, oy in the plane R, the third axis oz is then perpendicular to R, and the equation of the fundamental plane I is $Z=i$ with respect to these axes. Any point m of the plane I has its coördinates of the form:

$$(m)\ \begin{cases} x=p+ai \\ y=q+bi \\ z=i \end{cases}$$

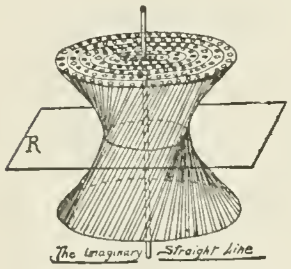
And it is evident that the real straight line M, whose equations are:

$$(M)\ \begin{cases} X=p+aZ \\ Y=q+bZ \end{cases}$$

passes through the given imaginary point m. It would be easy to show that the line M is the only real line passing through m, so that it is quite natural to take the line M as the representation of the point m in the plane I. By this method any imaginary plane curve drawn in I can be directly transformed into a congruence of real straight lines. In other words, by taking in the plane I, imaginary straight lines, circles, ellipses, hyperbolas, etc., through each point of these plane curves passes a real straight line in our space, and all these real lines will form congruences which we may call "*straight*," "*circular*," "*elliptic*," "*hyperbolic*," etc., congruences. Calling c an imaginary curve in I and C the corresponding congruence, c will be the focal surface of C, since each generator of C meets c. Moreover, any generator M intersects the plane $Z=i$ at the point m and the plane $Z=-i$ at the point n, whose coördinates are:

$$(n)\ \begin{cases} x=p-ai \\ y=q-bi \\ z=-i \end{cases}$$

Denoting the plane $Z=-i$ by J, it is seen that the real generator passing through m is the line joining m to its conjugate point in the plane J, and when m describes the curve c in I, its conjugate point n describes the conjugate curve c_{-1} in J, so that the congruence C is formed of the real generators obtained by joining each point of c to its conjugate point on c_{-1}; the two imaginary curves c in I and c_{-1} in J are then the two sheets or the two branches of the focal surface of the congruence C. In order therefore that a given congruence may be considered as the representation of an imaginary curve, it is necessary and sufficient that its focal surface be an analytic plane curve in the fundamental plane I. This fundamental plane will be supposed to be horizontal; the axis oz is then vertical, and any vertical straight line represents a *real* point of the plane I, since a and b vanish when the point becomes real. Moreover, the real point $x=p$, $y=q$, being represented by the vertical line $x=p$, $y=q$, when the given imaginary plane curve has a real part, this real part is represented by a vertical cylinder whose base is precisely the real part of the given curve.



The imaginary straight line: Let d be an imaginary straight line in the plane I, its conjugate curve d_{-1} in the plane J being also a straight line, the focal surface of the congruence D is composed of two straight lines; the congruence D is then a linear congruence; conversely when a linear congruence is analytic, it represents an imaginary straight line.

Taking the intersections of the plane I with the vertical planes xoz, yoz, as axes of coördinates in the plane I, and choosing the origin o such that oz meets the given imaginary line d, the equation of the latter will be: $Y=(m+ni)X$. If $x=p+ai$, $y=q+bi$ are the coördinates of any point m on d: $q+bi=(m+ni)(p+ai)$.

Or:
$$\begin{cases} b=ma+np \\ q=mp-na. \end{cases}$$

The real line M representing m is then:

$$(M)\ \begin{cases} X=aZ+p \\ Y=(ma+np)Z+(mp-na). \end{cases}$$

When a and p take all possible values, the line M generates the congruence D. The equations of M can be put under the form:

$$(M)\ \begin{cases} nX=n(aZ+p) \\ Y-mX=n(pZ-a). \end{cases}$$

Squaring, adding and putting $a^2+p^2=r^2$, gives:

$$(Y-mX)^2+n^2X^2=n^2r^2(Z^2+1).$$

When r is constant this equation represents an hyperboloid whose center is the origin and whose axis is the vertical axis oz. When $r=o$ the hyperboloid reduces to the vertical line oz; when r varies from o to ∞, the hyperboloids increase in diameter in such a way that their bases on the plane $Z=o$ are concentric and homothetic ellipses; the generators of these hyperboloids become more and more horizontal the greater r is.

In a word, the congruence representing an imaginary straight line looks exactly like a twisted bundle of rods; the central rod, being vertical, represents a real point; hence on any imaginary straight line there is a real point and only one which can be considered as the center of the imaginary line. If the same bundle of rods is now twisted in the opposite direction, we obtain the representation of the line: $Y=(m-ni)X$, which is conjugate of d; the central rod being common to both, shows that the intersection of two conjugate lines is a real point. It follows also that a system of two conjugate imaginary straight lines is the same thing as a vanishing real ellipse; for, being given a real ellipse, its real part is represented by a vertical elliptic cylinder, as remarked before; its imaginary part will be formed by an infinity of hyperboloids exterior to the elliptic cylinder and having the same vertical axis, so that when the real part of the ellipse shrinks into a point, what is left is evidently a system of two conjugate imaginary straight lines. If the given ellipse is a circle, the hyperboloids are of revolution, and when the circle shrinks into a point, it must reduce to two conjugate isotropic lines; hence the condition for an imaginary straight line to be isotropic is that the hyperboloids be of revolutions, *i. e.*, that the twisted bundle of rods be symmetric in all directions around its axis. Suppose d to be a real straight line $Y=mX+n$, m and n being real; then all the hyperboloids become infinitely flat, *i. e.*, their generators are all in the vertical plane $Y=mX+n$. The vertical lines of this plane are the real points, and the inclined lines the imaginary points of the given real line d.

Any plane curve can be studied in the same way; and the plane geometry can be transformed into a geometry of the ruled space. For instance, since two points determine a straight line, two generators are sufficient to determine a "straight congruence," and since the tangent to a curve is the straight line determined by two points of the curve infinitely near each other, a straight congruence will be said to be "tangent" to a given congruence when it has two generators infinitely near each other in common with it; since two points equally distant from a given point are on a circle drawn with the given point as a center, the "distance" of two generators in space will be the radius ($R=r_1+r_2i$) of the "circular congruence" passing through one of the generators and having the other as center, etc.

The axes ox, oy, oz, previously used, were only chosen to prove the first theorem and to study the properties of a "straight congruence." But, as remarked before, the method is not based upon any system of coördinates, and now that any imaginary straight line, imaginary distance and imaginary angle can be interpreted in the ruled space by real magnitudes, we can take two "straight congruences" perpendicular to each other as axes of coördinates, and define the position of any straight line M in space by its "distances" $x=\alpha+\beta i$, $y=\gamma+\delta i$ to the "congruences of coördinates," thus obtaining an analytic geometry of ruled space identical to the plane geometry in an imaginary plane.

René de Saussure.

German Manuscripts in the Library of Maihingen in Bavaria. By F. G. G. Schmidt.

During the past summer I had occasion to examine a number of German manuscripts in the library at Maihingen, belonging to Prince Karl of Öttingen-Wallerstein.

Maihingen, in the thirteenth century called Maigingen, Magingen, in the fourteenth, Maingen, Maigen, is a small village of about 700 inhabitants not far from the city of Nördlingen in the Bavarian district Schwaben-Neuburg.

The collection of manuscripts in Maihingen, numbering about 1500, is divided into five groups: the Oriental, the Greek, the Latin, the German and those of the Foreign Modern Languages (French, Dutch, etc.)

The oldest Codex among these manuscripts dates back to the sixth century. It is a New Testament in Anglo-Saxon script. Wattenbach has given an account of this manuscript in the Anzeiger des German. Museums, October, 1869.

Noteworthy also is a Greek commentary of the eleventh century of Andreas of Cæsarea, on the Apocalypse, which Erasmus used in editing the same. Here again, I may refer to Professor Delitzsch, Handschriftliche Funde, 1861.

The importance of the Maihinger Nibelungen manuscript (cod. Maih.) has been emphasized by Professor Zarncke.

A fragment of Notker's Psalms, also found in Maihingen, has been published by Hattemer. Other material has been brought before the public by R. von Liliencron, Meyer, Quen, and Bartsch.

In the year 1862 Professor Karl Bartsch visited his friend, the former librarian of Maihingen, Baron Freiherr von Löffelholtz. Through him he became acquainted with a portion of the contents of this library.

In Pfeiffer's Germania VIII, 48 f., K. Bartsch described a number of German and French codices seen by him during his short stay. But since the year 1862 different arrangements have been made in the library. The manuscripts at that time were not yet catalogued. It was undoubtedly this circumstance which prevented Professor Bartsch from seeing those of which I here give a brief account.

Most of the manuscripts examined by me—twenty-eight in number—belong to the fifteenth century, a few to the thirteenth and fourteenth centuries, and two to the seventeenth century. The majority of them have not yet been published, and by far the larger number have no duplicates at other libraries. Among these, the more interesting are the following: A fragment of 12 leaves in folio with the superscription: Das Lied von der heiligen Margaret, published by Professor K. Bartsch in Germanist. Studien, Vol. 1, Wien, 1872. According to Bartsch the author of this Margaret legend is a certain Wetzel or Wernher, nobleman and friend of Rudolf von Ems, who writes of this Wetzel in his "Alexander," in which he sings of his predecessors and contemporaries:

"Sante Margarêten leben
hât vil gefuoge gegeben
mîn friunt her Wetzel, des gih ich."

There exist many Margaret legends from the 12–15th century, which owe their popularity, no doubt, to the fact that the devil is so frequently represented as the hero. The language is that of the courtly lyrics of the thirteenth century, showing some influence of the Alemannic dialect.

The next manuscript in folio, likewise belonging to the fifteenth century, interests us less in regard to its contents than in regard to its author. It is entitled: *Petrus de Crescentiis vom Feld und Hausgemache*, comprising 128 leaves. It begins: Das ist das Register des ersten Buches Meyster Peters von Crescencien den walen und Burger von Banonien.

According to the notes in this manuscript, Petrus de Crescentiis was a philosopher in Bologna. He studied medicine, art and science, and travelled for more than twenty-five years in different countries. Later in life he studied law and held a justiceship in different Italian cities for many years. At the age of seventy he retired to the country, read books and wrote on agriculture. In the Italian language he wrote "Ruralium Commodorum libri duodecim," to please King Charles II, of Jerusalem and of Sicily. This work was first published in Augsburg, and was translated into German and French as well as Latin. He died in the beginning of the fourteenth century. A short account of his life is also given in Jöchers Allgemeines Gelehrten-Lexicon.

The following manuscripts, contained in the same volume, are probably by the same author: "Tractat vom Pflanzen der Bäume: liber [de] plantatione arborum," and: "Das Buch von der Haussorge: liber Oeconomicorum Aristotelis." This is only fragmentary. As to its contents, a closer examination reveals the fact that the manuscript has nothing in common with Aristoteles except the name and a few sentences.

In a large Latin codex in folio, I found a German manuscript of the fifteenth century: Postille, Winter und Sommerpredigten. The language of these sermons is of the thirteenth century. The author is not named, but there is hardly any doubt that we may identify him with Berthold of Regensburg.

Another manuscript gives an account of an earthquake in Italy in the year 1456. It consists of only one page, is written on parchment and was found in a Latin codex containing ecclesiastical and patristic matters from the eleventh to the fifteenth centuries, such as: Martyrologium Benedictinum; Epistola Aromatii et Eleodori ad Hyeronymum; Regula S. Benedicti, etc.

The smallest manuscript I found is a 'Trinklied.' The song is put to music: "Trinck, trinck, trinck, suff, suff, suff, bib, bib, bib, lauda bonum vinum—Schenk in wein mir und dir und dem, mir und dir und mir und dir und mir und dem."

Of more importance is an historical song in a volume of the fifteenth century. It is entitled: Wie Künig lassla zu prag in beham ward vmbgebracht (l. 30a–31b). R. von Liliencron published this song in 1865 in Volume I (No. 107) of his work: Die historischen Volkslieder der Deutschen vom 13–16. Jh. The printed song does not correspond in all respects with the copy made by me. Von Liliencron has omitted a small Latin appendix added to the song and referring to it. The author of the song, who informs us of the death of King Ladislaus in Prague, is a certain Knight Hans Wispeck at the Court of Vienna.

A paper manuscript in quarto, of 404 leaves, likewise belonging to the fifteenth century, contains three divisions:

The first, comprising pages 1a–124b, is entitled: Das Büchlein von der Liebhaltung Gottes. The material contained in this division is not worth dwelling upon.

The second division is: Geistliche Betrachtungen eines Karthäusers, l. 124b–352b.

In the introduction to these spiritual contemplations, a Carthusian monk, who signs himself Iohes K., requests a certain brother Kunrade to read over carefully this book written by him. At the conclusion of his introductory remarks he tells his brother Kunrade that he was acquainted with the celebrated Nicolaus von Dinckelsbühl, when he was a student at Vienna.

Jöcher, in his Allgemeines Gelehrten-Lexicon, gives the following account of the life of this Nicolaus von Dinckelsbühl: Nicolaus, a Swabian, and Augustinian hermit, was born in 1370 at Dünckelspiel, studied philosophy and theology at the University of Vienna. In 1405 he was elected Rector of this University, and at the same time Canonicus of the Church of St. Stephen. He remained at the University till 1416 as professor. In the same year he was sent to the council of Costnitz by the Archduke Albert V and by the University. After that he spent some time in the famous cloister of Melk, where he lectured to the young monks.

Melk has given a number of famous men to the world. I need mention only the two brothers, Bernhard and Hieronymus Pez, the first of whom published the Wessobrunner Gebet (1721) and valuable Old High German glosses, the other, in 1745, Österreich. Reimchronik des Ottokar.

In 1424 Nicolaus went as an ambassador of Albert V to Rome. He died in the cloister of Marienzell in 1433. His writings are published in Latin. Some of his manuscripts not yet printed are found in the University Library of Leipzig.

On page 136a begin a number of passages, of which I shall mention only the one on page 254a–301b. It is superscribed: Von ainer säligen Junckfrawen genannt elspethn. Amongst the many chapters this is the most interesting and most important.

In Volume IX, page 275 f., of the Alemannia we find under the title "Leben hl. alem. Frauen des 14/15 Jh." the following copy: Dit erst büchlyn ist von der seligen Klusneryn von Rütly, die genannt waz Elizabeth, published by Dr. Anton Birlinger, Professor of the University of Bonn.

After giving the text of this legend, Professor Birlinger states that this 'vita' is followed by those of the Klosterfrauen of Katharinenthal and by those of Töss. As the Alemannic version of our legend could not be found, the Hessian version is given according to a manuscript from Strassburg, which was discovered by Dr. Barack. Volume X brings the necessary literary historical biographical information, with a full description of the manuscript. Dr. Mone is said to have seen an Alemannic manuscript in Innsbruck, but it seems to be lost. In Volume X (page 128) of the Alemannia, Birlinger gives then the version of the Innsbruck manuscript of the life of the Klausnerin von Reute, and on page 88 f. the promised literary and critical notes on the German manuscript of 1624 and on the Latin manuscript of the fifteenth century. This copy Birlinger obtained from Dr. Oswald Zingerle, of Innsbruck. He found his conjecture that the dialect is Swabian-Augsburgian substantiated. Then follows the vita of the Dominicanerinnen von Kirchberg, near Haigerloch, and that of the nuns of Töss and Katharinenthal. Mention is furthermore made of the "Beta

Bona" in a sermon of Iakob Stehele in Wangen in the Allgäu, anno 1625; Violae Sanctorum, Ravensburg, by Jos. Schrötern, 1626, 4° s. q. And again in a manuscript, Kronik des 30j. Krieges im Breisgau, probably by a clergyman Mallinger von Basel-Freiburg; Merk, Pfarrer von Sigmaringen is quoted, who went with his patron to Reute. Neither note contains anything new.

According to these statements the Maihingen manuscript is up to the present time entirely unknown. There is little doubt, accordingly, that I have been so fortunate as to discover the oldest manuscript of the life of Elizabeth, the nun of Reute. It is certainly a version which must have followed closely the original, because the dialect is entirely Alemannic. It is the manuscript which has been missed for so many years. The writers of the two manuscripts known as the Strassburger and Innsbrucker have based their copies on the German original of the fifteenth century; so did the copyist of the manuscript from the year 1624. It is not entirely impossible that the Maihingen manuscript is the original. The report that the original manuscript was burnt in 1653 appears never to have been confirmed. There is another report that the original was destroyed by fire as early as 1633. But these assertions are not based on sound historical evidence.

My time did not allow me to continue my investigation. I therefore pass on to the third division, which is contained in the above mentioned manuscript. It is entitled: Geistliches Gespräch zwischen einer Fürstin und einer Kramerin von einem Paternoster aus Edelsteinen (l. 353^a–404^b.)

This book, as we read in the introduction, was written by Carthusian monks of the cloister Güterstein in the year 1447, and presented by them to a princess of Aurach in Württemberg.

Aurach, now Urach in literary German, while in Swabian dialects still called Aurach, and Güterstein have had a history which would be worth dwelling on. Güterstein, formerly called, ad bonum lapidem, zum guten Stein, was the cloister in which Rudolph von Ehingen closed his busy life, of which G. Schwab gives us a short account in his Neckarseite der Würtenbergichen Alb, p. 90.

On the inside of the front cover of this manuscript, containing these three mentioned divisions, the following lines are written: Das Buch gehört gen Kirche (dialectically still called so, now Kirchheim) zu Gebrauchs madalenen von Öttingen Eptissin daselbs und ist frav Agnessen von Werdenberg —gewesen ire mutter selig.

As to the contents of this dialogue, it is interesting to notice the Protestant views which sometimes break forth in spite of the Catholic subject which these monks had chosen. And besides, we have to keep in mind that this book was written long before the Reformation. Passages occur in this commentary on the pater noster which have a striking similarity with passages found in Zwingli's and Luther's Catechisms, written almost a century later. A large portion of the manuscript is devoted to a discussion on precious stones, such as the "saphir," "corneol," "jaspis" and others, and their magic power. Much attention is given to the "jaspis," the precious stone through the power of which the Phœnix renews himself out of the ashes, the stone which was said to be from the crown of Lucifer, and of which the dish in the Gralsage of Parzival is made. (Cf. Volume II of Schade's Altdeutsches Wörterbuch.)

An Early German Edition of Æsop's Fables. By George C. Keidel.

Among the more valuable books of the large collection bequeathed to the Johns Hopkins University by the late John W. McCoy is an edition of *Æsop's Fables* translated into German by the celebrated Dr. Hainricus Stainhöwel. The *editio princeps* of this collection of fables appears to be that printed at Ulm by Johannes Zeiner about the year 1475, a folio volume of 288 leaves, containing both the Latin text and Stainhöwel's German translation. This work was frequently reprinted during the fifteenth century, and the edition here described is undoubtedly a reprint of the German text alone, a policy first instituted it seems by Guentherus Zainer in his folio edition of 167 leaves, printed probably at Augsburg about 1480. There were also other editions of the same German text by various printers, and hence the most that can be claimed for the present one is that it is the oldest edition whose date is certain.

M. Léopold Hervieux states[1] that he has seen but two copies of this edition, one of which is in the private library of the King of Würtemburg at Stuttgart, and the other in the public library of Linz (Austria), where it is numbered D. iv. 9. According to his description the book is a folio of 169 leaves, of which the *Life of Æsop* and the text of the fables occupy the first 154 leaves, while the remainder contain a work entitled: *Historia Sigismunde.*

The McCoy copy is unfortunately not entirely complete, though the lacunæ are of no great extent. The first 34 leaves contain the *Life of Æsop* already mentioned and preceded by a full-page portrait headed: *Esopus.* This portion appears to be complete, except for the fact that the portrait in question, as well as the first five leaves of the text, has suffered a partial loss in its lower corner, apparently due to excessive thumbing. There then follow 120 numbered leaves containing the text of the fables, but of this series the fifth and sixth leaves are missing. Finally there comes a series of only eight additional numbered leaves, containing a table of contents, a portion of the *Historia Sigismunde* and the printer's colophon. The next to last leaf breaks off abruptly thus:

> sy inwendigen allein dÿe thür auf vnnd nam alldo—

At the top of the recto of the last leaf there occurs a colophon worded thus:

> Esopus der hochberümbt fabeltichter—mit
> etlichen zuogelegten fabeln Rimicy vund
> Auiani—vnd d' histori sigismunde der toh-
> ter des fürsten Tancredi vnd des iünglin
> ges Gwisgardi enndet sich hie—Gedruckt
> vnd vollendet in der hochwirdigen vnnd
> keiserlichen stat Augspurg—von Antho-
> nio Sorg am montag nach Agathe Da
> man zalt nach Cristi geburt—M—CCCC
> vnd in dem—LXXXIII—Iar—

The remainder of the leaf is blank, and on its verso there is written in pale black ink the name Johannes Schaffhaüsser, probably one of the early owners of this copy.

The present size of the leaves is about seven inches by ten, the type used is the Gothic, and the whole work is adorned by numerous rudely executed wood-cuts. The normal number of lines on full pages appears to be 36, although some have only 35; it is also to be noted that the following leaves are wrongly numbered:

leaf xii is given as xiii;
leaf liiii has no number;
leaf lvi is given as li;
leaf xci is given as ci; and
leaf cxv is given as cv.

A note in pale black ink on the upper margin of leaf xvi gives evidence of trimming by the binder, who appears to have greatly reduced what was originally a wide margin. The verso of this same leaf has had an extra illustration pasted over the one originally printed in the text, and as the superimposed wood-cut suits the accompanying text, yet appears to be wholly different from the one beneath,[2] though in the same style as the remaining illustrations, we may suppose that the printer erroneously inserted an irrelevant wood-cut in his text at this point, and discovering this fact after the leaf was printed, struck off special copies of the proper illustration and thus corrected his original error as well as he could. It would be of interest to note whether the same thing was done in the case of the other two copies mentioned above.

Many of the illustrations have been touched up with either black or red ink, and various marginal notes and other marks are to be found which are evidently due to some one or more of the early possessors of this rare old book.

A point worthy of note, and one which would probably suffice to easily identify all the extant copies of this edition, is that in certain cases a blank space has been left in the body of the text which should have been filled out by some word not inserted by the printer. Thus among the unnumbered leaves at the beginning of the book there is a blank space in the last line on the verso of the twenty-first leaf, and on the recto of the second numbered leaf there are three such spaces, the first of which has been filled in with a pen, the second crossed out, and the third left blank. These are the only cases of blank spaces which I have been able to find, and they constitute perhaps the most curious feature in the whole book.

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[1] *Les Fabulistes Latins*, Vol. 1, pp. 357–358; 2d ed., pp. 394–395. See also Brunet, *Manuel du Libraire*, 5th ed., Vol. 1, col. 101.

[2] This illustration is no doubt the same as the one correctly inserted in the text on the recto of leaf li.

An Explanation of "mio trapo" in the Poema del Cid, l. 2788. By C. C. Marden.

(Abstract of a paper read before the University Philological Association, November 15, 1895.)

> Despertedes, primas, por amor del Criador:
> Mio trapo [?] es el dia, ante que entre la noch,
> Los ganados fieros no nos coman en aqueste mont!

According to Vollmöller[1] the reading of the doubtful passage is *mio trapo* or *mie trap̄a*, the *ra* being expressed by a line of abbreviation after or over the *t*. The object of the present paper is to give an interpretation which, while being in harmony with the context of the poem, will not do violence to the passage as it is preserved to us in the original manuscript.

Hofmann's reconstruction *mietad pasó el dia*,[2] necessitates the introduction of *d* and the change of *es* to *so*. Cornu justly regards this reading as "quelque peu recherché" and suggests *mientra que*.[3] A study of the printed text shows that of the seventy-nine cases where the scribe has left out portions of a word, thirty-four show omission of syllables or letters other than *n*; forty-five show omission of *n*. The letter which shows the next largest number of omissions is *s*, and this mistake occurs only eight times. Hence we may conclude that if a letter is omitted after *mie*, that letter is most probably *n*. To get the *ra* Cornu naturally translates the line of abbreviation, and it is only in his change of *po* or *pa* to *que* that we meet with difficulty. In order to get the form *que*, we must alter *p* to *q*, introduce *u*, and change final *o* or *a* to *e*; if we suppose an abbreviated writing *q̄*, *p* must be changed to *q* and the final *o* or *a* is not accounted for. In either case the difficulty is increased by the fact that *p* and *q* are not easily confused in Old Spanish paleography.

Let us suppose that the final syllable of *mie trapa* is *ra* instead of *pa*. There is a strong resemblance between the letters *p* and *r* in the *Poema del Cid*, as can be seen from the fac-simile published by Rios,[4] and the mistaking one for the other could more readily occur where the scribe has prolonged the perpendicular stroke of the *r* below the line, as often happens in fourteenth century script. In the present instance a confusion is not unnatural since the passage under consideration has been rendered almost illegible by use of reagents.

We may now believe that the scribe first wrote *mie[n]tra*, using the abbreviation for *ra*; then seeing that such a form for this word was unusual, he immediately wrote the syllable *ra* in full. The horizontal bar over the *p*(*r*) may be the line used for scratching out the discarded abbreviation, or if the bar is a "drückfehler," the scribe forgot to erase the superfluous mark, —— just as he forgot to erase the superfluous letter in to(s)dos (l. 1832), fi(s)ta (l. 446), etc. If Völlmoller's *ra* is a horizontal line instead of a curve, this line may be the regular abbreviation for *n* which the scribe intended to place over the *e* of *mie*. Accepting either of these explanations, the verse becomes *Mientra es el dia, ante que entre la noch*, and the use of *mientra* instead of *mientra que* is justified by the occurrence of the shorter form in lines 925, 1047 of the same text.

RECENT PUBLICATIONS IN HISTORY AND POLITICS.

The following list enumerates publications, within the past year, in the department of History and Politics, by those who are now or who have been members of the University. It is printed for the information of their friends and colleagues and is not supposed to be complete.

Adams, Henry C. Annual Report on the Statistics of Railways in the United States. (*Washington: Interstate Commerce Commission*, 1895.)

—— Report of American Historical Association for the Year 1894. (*Washington*, 1895.)

Adams, Herbert B. Freeman the Scholar and Professor. (*Yale Review, Nov.*, 1895.)

—— Lord Amherst. (*New England Magazine, Feb.*, 1896.)

—— Report of the Eleventh Annual Meeting of the American Historical Association. (*Independent, N. Y., Jan.* 2, 1896.

—— Science and Good Government. (*Baltimore Sun, Nov.* 11, 1895.)

Bassett, J. S. The Cultural Opportunity of the Country Preacher. (*S. C. Christian Advocate, April* 11, 1895.)

—— Joseph Halstead Gillespie. (*Trinity Review, Oct.*, 1895.)

—— Landholding in Colonial North Carolina. (*Law Quarterly Review, April*, 1895.)

—— The Regulation and its Relation to the Revolution. (*Mid-Continent, July*, 1895.)

—— Frederick W. Robertson. (*Methodist Review, Nov.-Dec.*, 1895.)

—— The Study of History and Political Science for Southern Youths. (*Ib., Jan.-Feb.*, 1895.)

—— Suffrage in the State of North Carolina. (*Report of Am. Historical Association for* 1895.)

Bemis, E. W. Discussion of Labor Problems. (*Dial, Chicago, June* 16, 1895.)

—— A Point of View. (*Bibliotheca Sacra, No. CCIX.*)

Bentley, A. F. Units of Investigation in the Social Sciences. (*Annals of American Academy of Political and Social Science, Vol. V, No.* 6.)

Blackmar, F. W. Experiments on the Solution of the Labor Problem. (*Kansas University Quarterly, July*, 1895.)

—— History of Suffrage in Legislation in the United States. (*Chautauquan, Oct.*, 1895.)

—— Pensions in Legislation. (*Ib., Dec.*, 1895.)

Browne, William H. (Editor.) Archives of Maryland: Proceedings and Acts of the General Assembly, April, 1684–June, 1692; The Calvert Papers, No. 2. (*Baltimore*, 1895.)

Callahan, J. M. Agreement of 1817. Reduction of Naval Forces upon the American Lakes. (*Report of Am. Historical Association for* 1895.)

Carver, T. N. The Ethical Basis of Distribution and its Application to Taxation. (*Annals of American Academy of Political and Social Science, July*, 1895.)

Chambers, H. E. Editor of the Louisiana School Review. (*Begun April*, 1895.)

—— The Constitutional History of Hawaii. (*Baltimore, Johns Hopkins University Studies*, 1896.)

—— The Factors of Social Progress as Exemplified in the History of the Ancient Greeks. (*School Review, Monroe, La., April*, 1895.)

—— The Louisiana Educational Association and its Relation to the Educational History of the State. (*Address at Convention of La. Public School Teachers, April*, 1895.)

Chambers, H. E. Present Economic Conditions in the South. (*New Orleans Picayune, March* 11, 1895.)

—— Search Questions in American History. (*New York*, 1895.)

Commons, J. R. Progressive Individualism. (*American Magazine of Civics, June*, 1895.)

—— State Supervision for Cities. (*Annals of American Academy of Political and Social Science, Vol. V, No.* 6.)

Dewey, D. R. Irregularity of Employment. (*American Economic Association, Vol. IX, Nos.* 5, 6.)

Dewey, John. The Influence of the High School upon Educational Methods. (*School Review, Chicago, January*, 1896.)

Emmott, G. H. Address at First Meeting of the Lake Mohonk Conference on International Arbitration. (1895.)

—— An Arbitration Treaty between Great Britain and United States. (*Arena, Aug.*, 1895.)

Finley, R. J. The Cartoon in Politics. (*Review of Reviews, Dec.*, 1895.)

Gould, E. R. L. The Housing of the Working People. (*Report of the U. S. Commissioner of Labor*, 1895.)

Haynes, G. H. Representation in New England Legislatures. (*Annals of the American Academy of Political and Social Science, Sept.*, 1895.)

Haynes, John. A Cure for the Gerrymander. (*American Magazine of Civics, Aug.*, 1895.)

—— Review of Ingram's History of Slavery and Serfdom. (*Annals of the American Academy of Political and Social Science, Jan.*, 1896.)

—— Risk as an Economic Factor. (*Quarterly Journal of Economics, July*, 1895.)

Hazen, C. D. The French Revolution as seen by the Americans of the Eighteenth Century. (*Report of the American Historical Association for* 1895.)

Hendren, S. R. Government and Religion of the Virginia Indians. (*Johns Hopkins University Studies*, 1895.)
Dissertation for the Degree of Doctor of Philosophy.

Herriott, F. I. Review of Shaw's History of Currency. (*Annals of the American Academy of Political and Social Science, Jan.*, 1896.)

Hollander, J. H. Some Unpublished Letters of David Ricardo. (*Quarterly Journal of Economics, January*, 1896.)

Hunt, R. D. The Genesis of California's First Constitution, 1846–49. (*Johns Hopkins University Studies*, 1895.)
Dissertation for the Degree of Doctor of Philosophy.

Ingle, Edward. Literary Messenger. The Leading Magazine of Dixie before the War. (*Richmond Times, Nov.* 3, 1895.)

—— Poe as a Critic. (*Richmond Dispatch, Oct.* 27, 1895.)

James, B. B. The Law of Diminishing Returns and Ecclesiastical Development. (*Methodist Protestant, Baltimore, Jan.* 15, 1896.)

James, J. A. Can there be too many College Graduates? (*Proceedings Iowa State Teachers' Association*, 1895.)

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[1] *Poema del Cid*, Halle, 1879.

[2] *Zts. für Rom. Phil.*, IV, p. 159.

[3] *Romania*, XI, p. 93.

[4] *Hist. de la Lit. Esp.*, III.

Johnson, E. R. Industrial Services of the Railways. (*Annals of the American Academy of Political and Social Science, May*, 1895.)

—— Railway Departments for the Relief and Insurance of Employees. (*Ib.*, 1895.)

—— The Nicaragua Canal and the Economic Development of the United States. (*Ib., Jan.*, 1896.)

Jones, C. E. Address before the Confederate Survivors' Association of Augusta. (1895.)

Lee, G. C. Negroes under Northern Conditions. (*Gunton's Magazine, Jan.*, 1896.)

Levermore, C. H. The Whigs of Colonial New York. (*American Historical Review, Jan.*, 1896.)

Lynes, G. B. Seeley, the Historian. (*Baltimore American, June* 9, 1895.)

—— Union College. The Celebration of its First Centennial. (*N. Y. Tribune, June* 23, 1895.)

Mikkelsen, M. A. Cultivation of Vacant City Lots by the Unemployed. (*Independent, May* 9, 1895.)

—— Lincoln's Works. (*Outlook, March* 16, 1895.)

Minter, C. W. Protest against the Evils of the Depreciated Continental Currency, entered on the Essex County "Probate Records" in 1781 by the then Probate Judge. (*Quarterly Journal of Economics, Jan.*, 1896.)

Moore, H. L. Von Thünen's Theory of Natural Wages, I–II. (*Quarterly Journal of Economics, April and July*, 1895.)

Ramage, B. J. Sunday Legislation. (*Sewanee Review, November*, 1895.)

Reitzenstein, Milton. The Walters Art Gallery. (*New England Magazine, July*, 1895.)

Scaife, W. B. Popular Civic Education in France. (*Public Opinion, May* 9, 1895.)

—— The Public Schools of Geneva. (*Educational Review, Dec.*, 1895.)

—— What is the Monroe Doctrine? (*Pittsburg Dispatch, Dec.* 13, 1895.)

Schouler, James. Historical Testimony. (*Report of the American Historical Association for* 1895.)

Seager, H. R. The Seventh Annual Meeting of the American Economic Association. (*Annals of the American Academy of Political and Social Science, March*, 1895.)

—— Review of Cunningham's Outlines of English Industrial History. (*Ib., Jan.*, 1896.)

Shaw, Albert. Recent Progress of Italian Critics. (*Review of Reviews, Nov.*, 1895.)

—— South Carolina's New Constitution. (*Ib., Jan.*, 1896.)

Shaw, W. B. Social and Economic Legislation of the Several States in 1895. (*Quarterly Journal of Economics*, 1896.)

Sherwood, Sidney. Money in Legislation. (*Chautauquan, Jan.*, 1896.)

Silver, J. A. The Provisional Government of Maryland (1774–1777.) (*Johns Hopkins University Studies*, 1895.)
Dissertation for the Degree of Doctor of Philosophy.

Small, A. W. American Society. (*Chautauquan, Oct.*, 1895.)

—— The Civic Federation of Chicago. (*American Journal of Sociology, Vol. I, No.* 1.)

—— Static and Dynamic. (*Ib., Vol. I, No.* 2.)

—— The State and Semi-Public Corporations. (*Ib., Jan.*, 1896.)

—— "Social" *vs.* "Societary." (*Annals of the American Academy of Political and Social Science, May*, 1895.)

Smith, C. L. Cuba and Cubans: A Chapter of Spanish Oppression. (*Central Baptist, Dec.* 19, 1895.)

—— Moslem *versus* Christian: The Persecutions in Armenia. (*Ib., Dec.* 12, 1895.)

Steiner, B. C. Citizenship and Suffrage in Maryland. (*Baltimore*, 1895.)

—— Cokesbury College. Read at Abingdon, Md., Oct. 18, 1895. (*Baltimore Methodist, Oct.* 31, 1895.)

—— Electoral College for the Senate of Maryland. (*Report of the American Historical Association for* 1895.)

—— Guilford's Early Libraries. (*Guilford (Conn.) Echo, Nov.* 9, 16, 1895.)

Stephenson, Andrew. A Step in the Transition from Communism to Private Property in Land. (*Proceedings of the Indiana College Association*, 1895.)

Thomas, T. P. City Government of Baltimore. (*Johns Hopkins University Studies*, 1896.)
Dissertation for the Degree of Doctor of Philosophy.

Trent, W. P. (Editor.) The Sewanee Review. (*Sewanee*, 1895.)

Tolman, W. H. Fresh Air Work in New York City. (*Chautauquan, Sept.*, 1895.)

Turner, F. J. Western State-making in the Revolutionary Era. (*American Historical Review, Oct.*, 1895; *Jan.*, 1896.)

Veblen, T. B. (translator.) Gustav Cohn's Science of Finance. (*University of Chicago*, 1895.)

Vincent, J. M. Review of Emerton's Middle Ages. (*Educational Review, June*, 1895.)

Weeks, S. B. Colonial Culture in North Carolina. (*Report of the American Historical Association for* 1895.)

—— Southern Slavery and the Quakers. (*Johns Hopkins University Studies, Extra Volume XV.*)

Wetzel, W. A. Benjamin Franklin as an Economist. (*Johns Hopkins University Studies*, 1895.)
Dissertation for the Degree of Doctor of Philosophy.

Williams, H. W. Money and Bank Credits. (*Annals of the American Academy of Political and Social Science, Jan.*, 1895.)

Willoughby. W. F. The Musée Social in Paris. (*Ib., Jan.*, 1896.)

Wilson, Woodrow. A Literary Politician. (*Atlantic Monthly, Nov.*, 1895.)

CURRENT NOTES.

Lectures have been recently given as follows:

Hon. Frederick Howard Wines, LL. D., of Springfield, Ill., gave six lectures on Social Problems, December 9–20, in McCoy Hall. The special topics were: The Genesis of Social Classes, The Classification of Social Evils, The Problem of Poverty, The Problem of Crime, The Duty of the State to Special Classes, Function of the University relative to Social Evils.

Professor Harold N. Fowler, of the Western Reserve University, gave an illustrated lecture on Greek Art in the Age of Pericles, in McCoy Hall, December 12.

Professor Davis R. Dewey, of the Massachusetts Institute of Technology, gave a course of twelve lectures, in the Donovan Room, on the Conditions and Remedies of Non-Employment, beginning January 21.

Professor Woodrow Wilson, of Princeton College, gave a course of twenty-five lectures, in the Donovan Room, on the Theory and Organization of Local Government, beginning January 23.

Professor N. S. Shaler, of Harvard University, gave an illustrated lecture in McCoy Hall, February 6, on Volcanoes.

The Franklin Institute of Philadelphia, has awarded the John Scott Legacy Medal for 1895 to Lieut. G. O. Squier, U. S. A. (Ph. D., 1893), and Albert C. Crehore (graduate student, 1890–91), for their Polarizing Photochronograph.

George W. Smith (graduate student, 1890–92) was inaugurated President of Colgate University November 14. President Gilman gave the address at the inauguration ceremonies.

Professor Rowland has been elected a Foreign Correspondent of the French Academy of Sciences, and a Foreign Member of the Italian Society of Spectroscopists. He has also been promoted to the rank of Officer of the French Legion of Honor.

Professor Newcomb has also been promoted to the rank of Officer of the French Legion of Honor, and has been elected a Member of the Academy of the Lincei at Rome.

Professor Brooks has been recently elected a Fellow of the Royal Microscopical Society.

UNIVERSITY SCHOLARSHIPS.

The Board of University Studies, at a meeting held January 8, awarded University Scholarships to the gentlemen named below:

H. L. Clark (A. B., Amherst College). *Biology.*
Pelham Edgar (A. B., University of Toronto). *English.*
A. S. Haggett (A. B., Bowdoin College). *Greek.*
J. G. Hardy (A. B., Lafayette College). *Mathematics.*
F. R. Jones (A. B., Western Maryland College). *History.*
P. H. Land (A. B., Calvin College). *Semitic Languages.*
W. T. Mather (A. B., Amherst College). *Physics.*
G. S. Maynard (A. B., Johns Hopkins University). *Electricity.*
O. F. Long (A. B., Kentucky Wesleyan College). *Latin.*
J. A. Ness (A. B., Luther College). *Sanskrit.*
W. A. Nitze (A. B., Johns Hopkins University). *Romance Languages.*
M. D. Sohon (M. S., Lehigh University). *Chemistry.*
A. C. Spencer (S. B., Case School). *Geology.*

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The vacancies in the Hopkins Scholarships offered to graduate students from Virginia and North Carolina have been filled as follows:

Honorary Scholarship:

William Francis Gill, of North Carolina.

Ordinary Scholarships:

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G. C. Lee, of North Carolina.
J. W. Reid, of North Carolina.

The Ordinary Hopkins Scholarships offered to newly entering matriculated students from Maryland, on competitive examination, have been awarded as follows:

Ralph Duffy,
Henry Wireman Cook,
Victor Edgeworth Smith,
Joseph Nathan Ulman,
Albert Gad Stidman,
Henry John Lucke.

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